Unit 2: Axioms and arrays

Week 1 Lesson 1: What is multiplication?

Concept Corner

Copy and complete:

3	four	4	inverse	4

Division is the _____ operation of multiplication.

In a multiplication/division fact family, there are _____ related calculations.

The diagram below shows that:

$4 \times 3 = 12$ $\quad 12 \div \_\_\_ = 3$ $\quad 3 \times \_\_\_\_ = 12$ $\quad 12 \div \_\_\_\_ = 4$

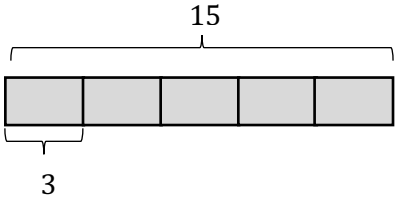
1. Use the bar model to copy and complete the calculations in the following fact family:

$5 \times 3 = \_\_\_\_$

$15 \div \_\_\_\_ = 3$

$3 \times \_\_\_\_ = \_\_\_\_$

$15 \div \_\_\_\_ = 5$



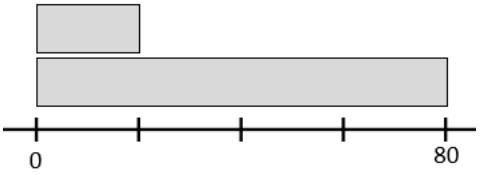
2. Copy and complete the bar model and fact family below:

$\_\_\_\_ \times 4 = \_\_\_\_$

$80 \div \_\_\_\_ = \_\_\_\_$

$\_\_\_\_ \times \_\_\_\_ = \_\_\_\_$

$\_\_\_\_ \div \_\_\_\_ = \_\_\_\_$



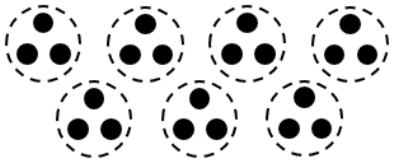
3. Draw the diagram below and complete the fact family that it represents.

$\_\_\_\_ \times 3 = \_\_\_\_$

$\_\_\_\_ \div 7 = \_\_\_\_$

$\_\_\_\_ \times \_\_\_\_ = \_\_\_\_$

$\_\_\_\_ \div \_\_\_\_ = \_\_\_\_$

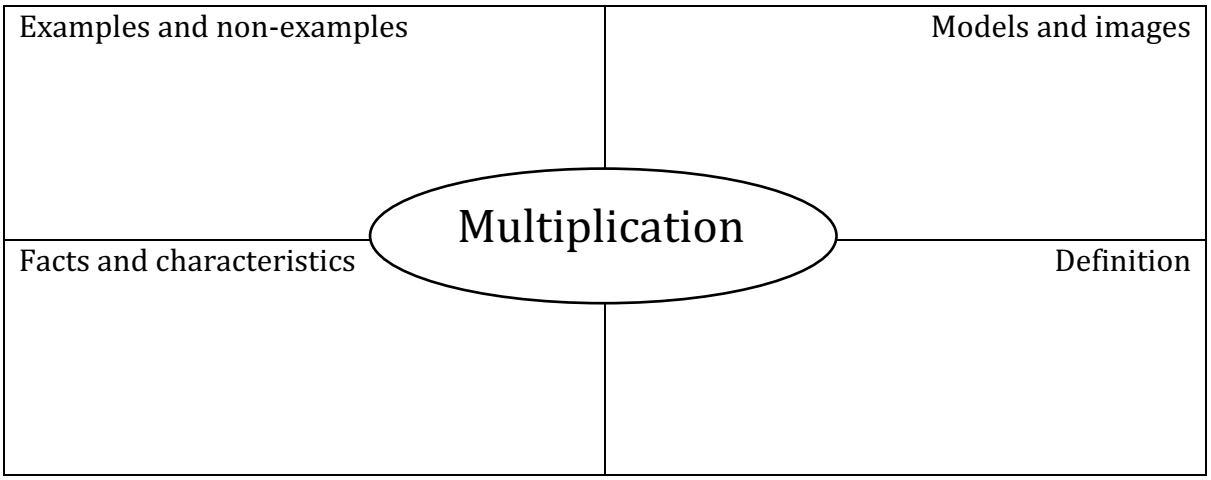


4. a) Write the four different calculations in the fact family of 5×6
   b) Draw three different diagrams to represent 5×6

5. For each problem, draw a model to represent and calculate the solution.

A bag contains 7 sweets. There are 9 bags. How many sweets are there in total?
A bag contains 9 sweets. There are 7 bags. How many sweets are there in total?
A bag contains 9 sweets. There are 63 sweets in total. How many bags of sweets are there?
A bag contains 7 sweets. There are 63 sweets in total. How many bags of sweets are there?
There are 63 sweets in total. They are put into 9 bags. How many sweets in each bag?
There are 63 sweets in total. They are put into 7 bags. How many sweets in each bag?

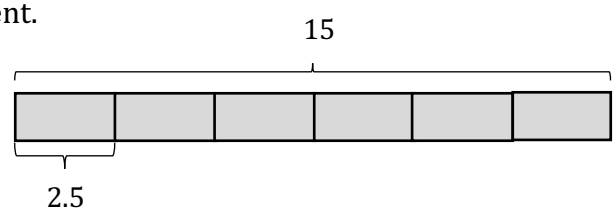
6. Copy and complete a Frayer model to describe multiplication



Questions for depth

1. Given that $2.5 \times 6.4 = 16$.
   Find the three remaining calculations in the fact family.

2. Write at least four different word problems involving multiplication or division that this model could represent.



Week 1 Lesson 2: What is commutativity?

Concept Corner

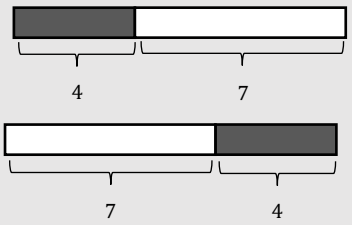
order	7	commutative	3

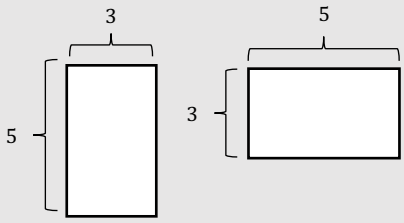
Copy and complete:

If an operation is **commutative** then we can apply the operation to two numbers in any _____.

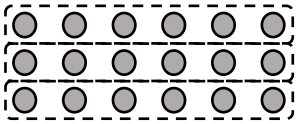
For example, addition and multiplication are _____:

e.g. $\mathbf{4 + \_\_\_ = 7 + 4}$ $\mathbf{3 \times 5 = 5 \times \_\_\_}$





1. Jennie has drawn an array, she has grouped the dots to show $3 \times 6 = 18$



   a) Draw the array, group the dots differently to show $6 \times 3 = 18$.

   b) By grouping an array in two different ways, show that $4 \times 3 = 3 \times 4$.

   c) What other calculations can you show using a 4×3 array?

2. a) Find two different ways to complete this calculation.

$$32 = \square \times \square$$

   b) How many other ways can you fill in the blanks in the calculation using positive integers?

3. Zara buys 20 red apples for 23p each.
   Charlotte buys 23 green apples for 20p each.

   a) How much did each person spend?

   b) If Zara bought 40 red apples how many green apples would Charlotte have to buy to spend the same?

4. Complete the calculations below to create three different fact families.
   Draw a diagram to represent each one

$$\_\_\_ \times \_\_\_ = 100$$
$$100 \div \_\_\_ = \_\_\_$$
$$\_\_\_ \times \_\_\_ = \_\_\_$$
$$\_\_\_ \div \_\_\_ = \_\_\_$$

5. Copy and complete the following multiplication grids.

a)

×	4			9
2		10		
	12		21	
11				
		60		

b)

×			11	
3		27		
	25			65
			66	
	40			

Questions for depth:

1. Akira notices an interesting pattern:

   a) What is the next equation in the list?

   b) Write a sentence to describe the pattern.

   c) Explain why this pattern will always work.

$$1 + 1 + 1 = 3$$
$$2 + 2 + 2 = 3 + 3$$
$$3 + 3 + 3 = 3 + 3 + 3$$
$$4 + 4 + 4 = 3 + 3 + 3 + 3$$
$$5 + 5 + 5 = 3 + 3 + 3 + 3 + 3$$

2. Tom wants to buy single scoop of ice cream with one topping.

Alice's Ice Cream Parlour

Flavours: *Chocolate or Vanilla*

Toppings: *Nuts, Chocolate Sauce, Sprinkles or a Flake*

Bill's Ice Cream Shop

Flavours: *Chocolate, Vanilla, Strawberry or Coffee*

Toppings: *Nuts or a Flake.*

   a) Does Tom have more options in Alice's or Bill's? Justify your answer.
   b) If instead Tom wanted two scoops and one topping, which would give him more choice? Justify your answer.

Week 1 Lesson 3: Multiplication and Division

Concept Corner

7	21	groups	equal

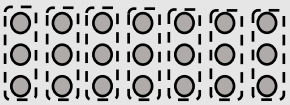
Copy and complete:

Division can be interpreted in different ways.

For example, $24 \div 6 = 4$ can be understood as 24 divided into 6 _____ groups or as 24 divided into _____ of 6.

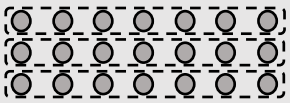
Understanding that multiplication is commutative allows us to be flexible with the way we think about division. Knowing that:

21 is equal to ___ groups of 3



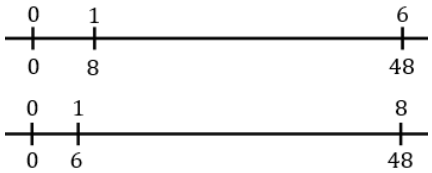
can be used to work out:

___ divided into 3 equal groups



1.

a) Explain how the following models show that $6 \times 8 = 8 \times 6$



b) Draw two similar models to show the following:

i. $2 \times 6 = 6 \times 2$

ii. $5 \times 7 = 7 \times 5$

2. Explain how the two division problems link to the multiplication fact $6 \times 4 = 24$

i. 24 biscuits are shared amongst 4 people. How many biscuits does each person receive?

ii. 24 biscuits are shared, each person receives 4 biscuits. How many people are there?

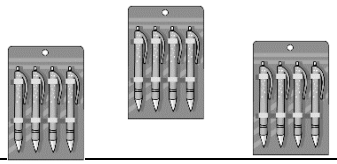
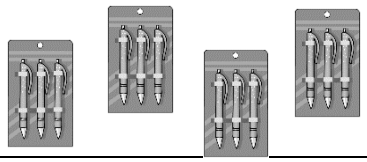
Write similar pairs of division problems for the following multiplication facts:

iii. $12 \times 6 = 72$

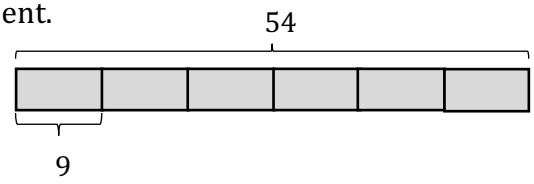
iv. $150 \times 6 = 900$

v. $\frac{1}{2} \times 8 = 4$

3. Look at the following table and write a statement for each empty cell.

		
3×4	Three packs of four pens is 12 pens in total	There are three pens per pack. There are four packs so 12 pens in total
4×3	There are four pens per pack. There are three packs so 12 pens in total	a)
$12 \div 3$	There are 12 pens in three packets. Each packet contains 4 pens	b)
$12 \div 4$	c)	d)

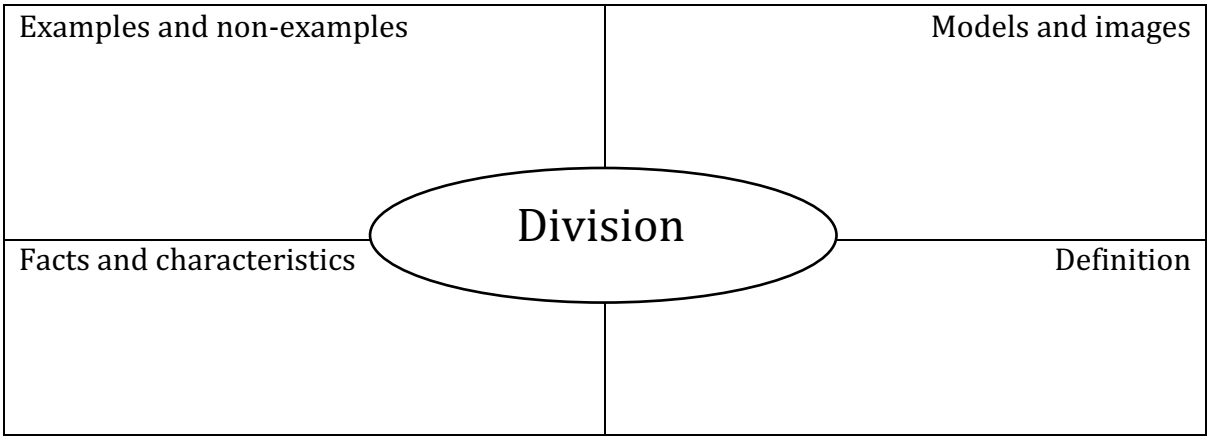
4. Write at least four different word problems involving multiplication or division that this model could represent.



Questions for depth

1. Explain how to use:
   a. $400 \div 25 = 16$ to work out $384 \div 16$
   b. $221 \div 17 = 13$ to work out $234 \div 13$
   c. $322 \div 14 = 23$ to work out $299 \div 23$
   d. Create some of your own examples of this style of question

2. Copy and complete a Frayer model to describe division



Week 1 Lesson 4: What is Associativity?

Concept Corner

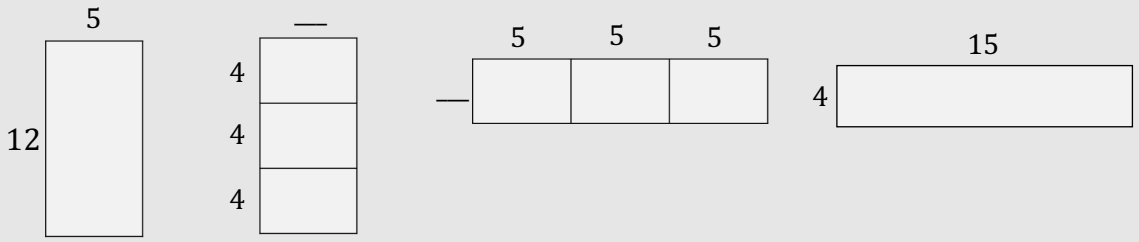
Copy and complete:

15	5	4	associativity	3	3

These calculations and diagrams show how _____ has been used to show that

$5 \times 12 = 15 \times 4$

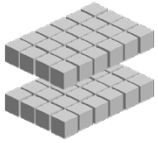
$$\mathbf{5 \times 12} \quad = \quad \mathbf{5} \times (\_\_ \times \mathbf{4}) \quad = \quad (\mathbf{5} \times \_\_) \times \mathbf{4} \quad = \quad \_\_ \times \mathbf{4}$$



1. Evaluate the expressions in each pair to show that they are equal:
   a) $(3 \times 8) \times 5$ and $3 \times (8 \times 5)$
   b) $(18 \times 2) \times 5$ and $18 \times (2 \times 5)$
   c) $(2.5 \times 4) \times 2$ and $2.5 \times (4 \times 2)$

2. Sally uses 48 cubes to make a cuboid. She breaks up the cuboid in four different ways. For each image, copy and complete the corresponding calculation.

a)

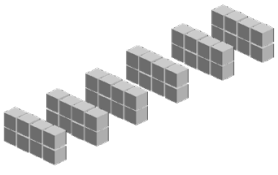


$2 \times (\_\_ \times \_\_)$

b)

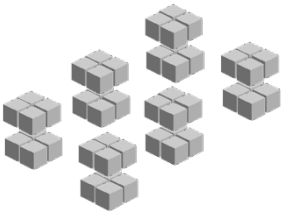
$4 \times (\_\_ \times \_\_)$

c)



$\_\_ \times (4 \times \_\_)$

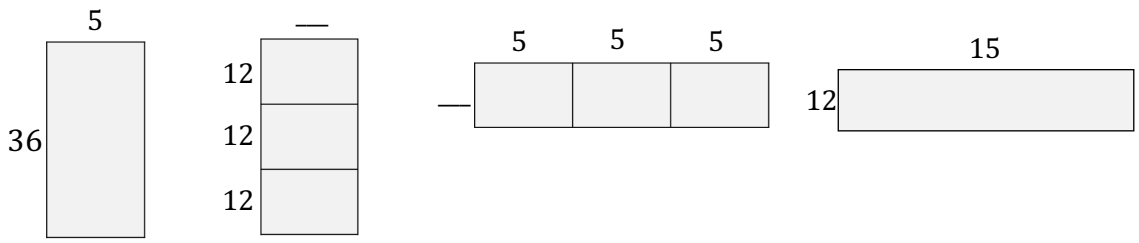
d)



$(\_\_ \times \_\_) \times \_\_$

3. Complete the calculations to match the diagrams:

$5 \times 36 \quad = \quad 5 \times (\_\_ \times 12) \quad = \quad (5 \times \_\_) \times 12 \quad = \quad \_\_ \times 12$



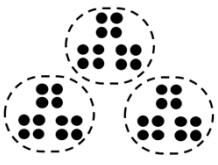
4. Copy and complete the calculations:
   a) $16 \times 5 = (8 \times \_\_) \times 5 = 8 \times (\_\_ \times 5) = 8 \times \_\_ = \_\_$
   b) $16 \times 35 = 16 \times (5 \times \_\_) = (16 \times \_\_) \times \_\_ = \_\_ \times \_\_ = \_\_$
   c) $25 \times 6 = 25 \times (\_\_ \times 3) = (25 \times \_\_) \times 3 = \_\_ \times 3 = \_\_$
   d) $25 \times 12 = 25 \times (\_\_ \times \_\_) = (25 \times \_\_) \times \_\_ = \_\_ \times \_\_ = \_\_$
   e) $8 \times 35 = 8 \times (\_\_ \times \_\_) = (8 \times \_\_) \times \_\_ = \_\_ \times \_\_ = \_\_$
   f) $1.6 \times 35 = 1.6 \times (\_\_ \times \_\_) = (1.6 \times \_\_) \times \_\_ = \_\_ \times \_\_ = \_\_$

5. Use the fact that $32 \times 28 = 896$ to work out the following calculations:
   a) $10 \times 32 \times 28$
   b) 32×280
   c) 64×28
   d) 16×56
   e) 3.2×2800
   f) 160×56

<u>Questions for depth</u>

1. For each diagram use the groups to write a calculation. The first has been completed.

e.g.

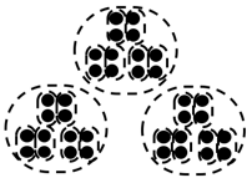


$12 \times 3 = 36$

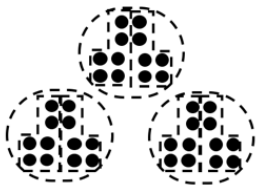
a)

b)

c)



d)



e)

2. Create a similar diagram to the question above for the 24 dots. Find different ways of writing 24 using the different groupings of dots.

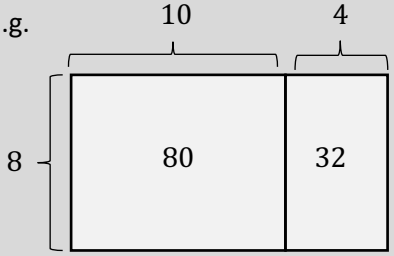
Week 2 Lesson 1: What is the distributive property?

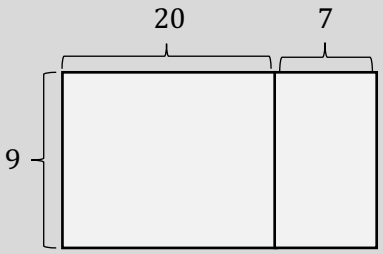
Concept Corner

Below is a calculation and diagram to show the **distributivity** of multiplication over addition.

Use the worked example to copy and complete the other diagram and calculation.

e.g.





$8 \times 14 = 8 \times (10 + 4)$

$= 8 \times 10 + 8 \times 4$

$= 80 + 32$

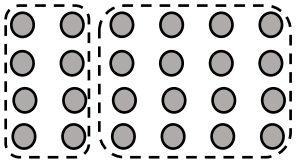
$9 \times 27 = \_\_ \times (\_\_ + \_\_)$

$= \ldots$

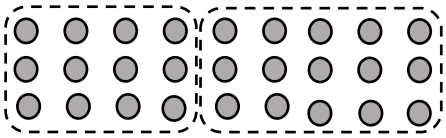
$=$

1. For each diagram write down the corresponding equation. The first one has been done for you.

a)

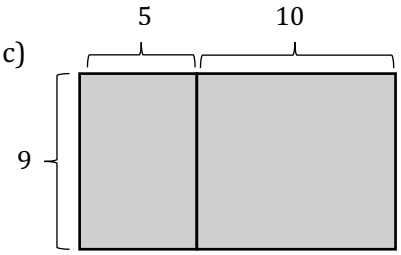


b)

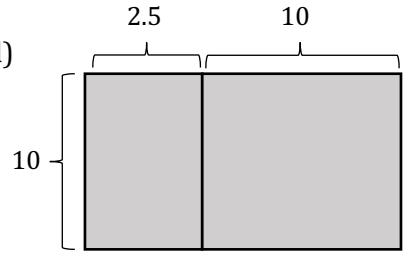


$4 \times (2 + 4) = 4 \times 2 + 4 \times 4$

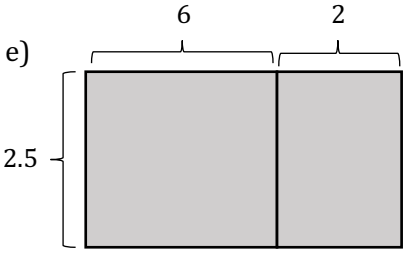
c)



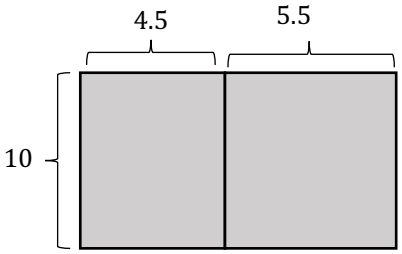
d)



e)



f)



2. Complete the different ways of using distributivity for calculating 5×17:

a) $5 \times 17 = 5 \times (10 + \_\_) = 5 \times 10 + 5 \times \_\_ = 50 + \_\_ = \_\_$

b) $5 \times 17 = 5 \times (8 + \_\_) = 5 \times 8 + 5 \times \_\_ = 40 + \_\_ = \_\_$

c) $5 \times 17 = 5 \times (5 + \_\_) = 5 \times 5 + 5 \times \_\_ = \_\_ + \_\_ = \_\_$

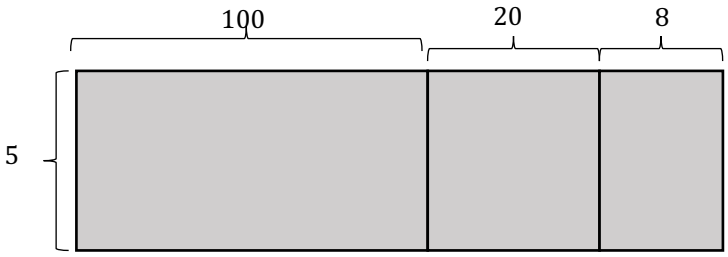
d) $5 \times 17 = 5 \times (20 - \_\_) = 5 \times 20 - 5 \times \_\_ = \_\_ - \_\_ = \_\_$

e) Which way is your preferred method? Why?

3. Use the distributive property to calculate each of these in two different ways. Write equations to show each step of your calculation strategy.

a) 5×24 b) 11×11 c) 8×99 d) 2.5×12

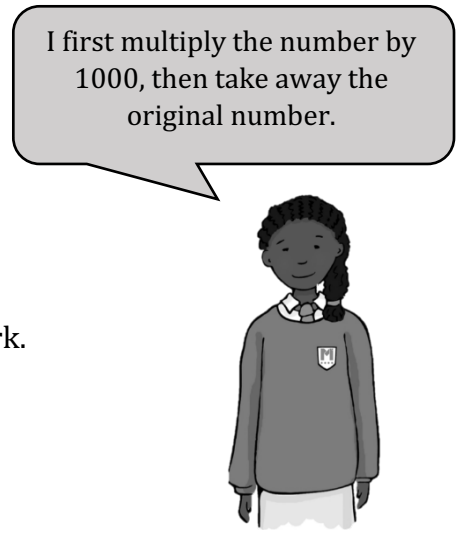
4. Arianna wants to calculate 5×128. She draws this diagram to help her:



a) Write a corresponding calculation represented by the array and find a solution.
b) Use associativity to help you to calculate 5×128 in a different way.
c) Draw a diagram to show how distributivity can be used to calculate 6×1328.

Questions for depth

1. Priya has a method for multiplying a number by 999.



a) Use Priya's method to calculate 7×999

b) Use Priya's method to calculate 999×999

c) Draw a diagram to show why this method will always work.

2. Adapt Priya's method to help you to calculate:

a) 9999×9999

b) 99999×99999

c) 999999×999999

Week 2 Lesson 2: Multiplication Tables

Concept Corner

Copy and complete:

Understanding of the axioms can be used to explain and understand relationships between _____ facts.

2	10	times tables
7	4	doubling

For example, associativity can be used to explain why values in the 4 times tables can be found by _____ the values in the ___ times tables.

$$(3 \times 2) \times 2 = 3 \times (2 \times 2) = 3 \times \_\_$$

Distributivity shows the relationship between the 2, 10 and 12 times tables.

$$7 \times 12 = 7 \times (\_\_ + 2) = 7 \times 10 + \_\_ \times 2$$

1. Copy and complete the calculations below:

a) $2 \times (5 \times 4) =$ ___

b) $7 \times 8 = 8 \times \_\_$

c) $9 \times 4 = 9 \times (2 \times \_\_)$

d) $64 = 4 \times \_\_ \times 8$

e) $\_\_ = 4 \times 12 + 8 \times 12$

f) $12 \times 6 = 2 \times (\_\_ \times 6)$

g) $6 \times \_\_ + 5 \times \_\_ = 7 \times (6 + 5)$

h) $88 = 4 \times (\_\_ \times \_\_)$

2. Give an example to show each of the following statements are true.
An example has been done for you:

e.g. If I know my 10 and 2 times tables then I can find my 8 times tables...

...for example, to find 8×7 *I can subtract* 2×7 *from* 10×7

a) If I know my 2 times table I can work out my 4 or 8 times tables
b) I can find my 9 times table from my 10 times table
c) I can find my 6 times table using the 5 times table
d) I can work out my 7 times table if I know my 5 and 2 times tables

3. Some numbers appear in many of the 1-12 times tables.
   For example, 16 appears in the 1, 2, 4 and 8 times tables.

   Look at the numbers below. Put them in order for how many of the 1-12 times tables they appear in (least to most).

   24 32 36

4. Given that $18 \times 13 = 234$, find the solutions to the following calculations:

   a) 9×13 b) 36×13 c) 9×26 d) 19×13

   e) 28×13 f) 18×6.5 g) 9×6.5 h) 13×18

5. Decide if the statements below are true or false, with examples to show when false.
   a) Every number in the 4 times table is also in the 2 times table.
   b) Every number in the 8 times table is also in the 12 times table.
   c) Every number in the 12 times table is also in the 3 times table.
   d) Every number in the 3 times table is also in the 6 times table.
   e) Every *even* number in the 5 times table is also in the 10 times table.

Questions for depth

1. The statements below refer to how many times numbers appear in a 1**-12 times tables grid**. Explain ***why*** they are all true:

   a) Prime numbers appear an ***even*** number of times.
   b) Square numbers appear an ***odd*** number of times.
   c) 24 appears ***more times*** than any other number.

2. The statements below refer to times tables that continue to infinity. Decide if the statements below are true or false.
   a) Only half of the numbers in the 6 times table are in the 3 times table.
   b) One quarter of the numbers in the 12 times table are in the 4 times table.
   c) Half of the even numbers in the 3 times table are in the 12 times table.
   d) One seventh of the numbers in the 7 times table are multiples of 3.

3. Create your own statements that could be included in questions 1 & 2.

Week 2 Lesson 3: Number pyramids

Concept Corner

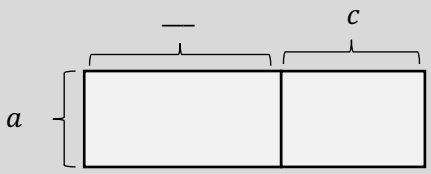
b *a* multiplication

Copy and complete:

The distributive property tells us an important relationship that involves _____, addition and subtraction.

We can represent the distributive property using diagrams and calculations. We don't need to know the values in the calculations, and can show the relationship using _____:

$a \times (b + c) = \_\_ \times b + a \times c$

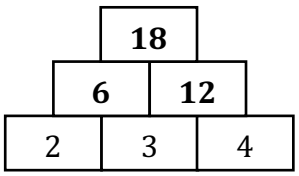


1. The bricks in the middle row of the pyramids are completed by multiplying the numbers in the two bricks beneath them.

   The top brick is completed by summing the numbers in the middle row.

   Copy and complete the pyramids below. The first one has been done for you.

a)



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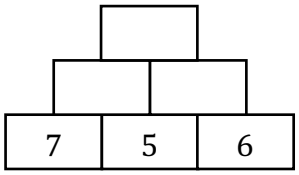
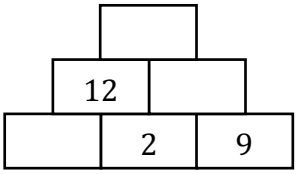
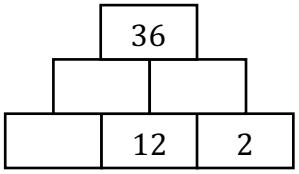


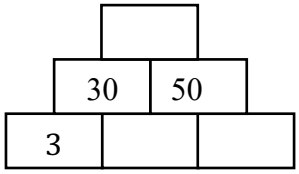
c)



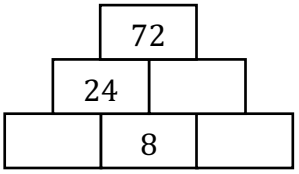
d)



e)

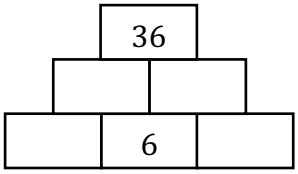


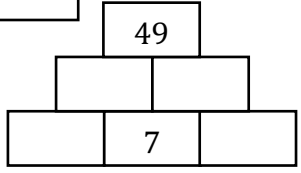
f)



2. The pyramids below have the same rules as the pyramids in question 1.
   a) Complete the pyramids in as many different ways as you can using positive integers.

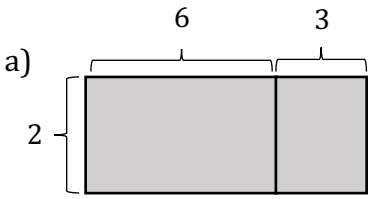
   b) What do you notice about **sum** of the outer bricks in the bottom row for each solution? How is this connected with the number in the top brick?



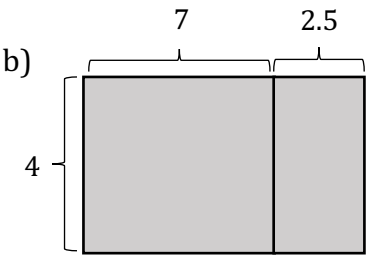


3. The diagrams below can be used to show the distributive property. For each diagram write out two equivalent expressions. The first has been done for you.

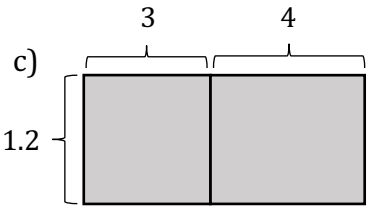
a)



b)

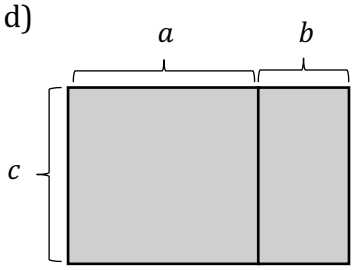


c)

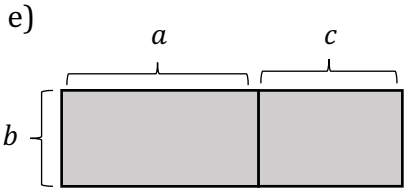


$2 \times (6 + 3) = 2 \times 6 + 2 \times 3$

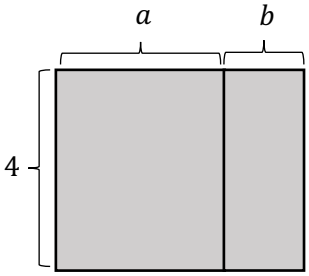
d)



e)



f)



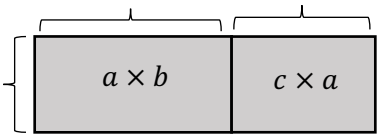
4. Match the calculations below into pairs.

$6 \times (7 + 8)$	$7 \times 8 + 6 \times 8$	7×14
$(6 + 8) \times 7$	$6 \times 7 + 8 \times 6$	$8 \times (7 + 6)$

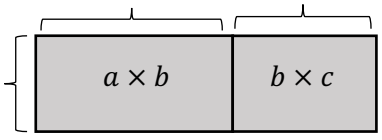
Questions for depth

1. Label the diagrams and write expressions that each could represent

a)



b)



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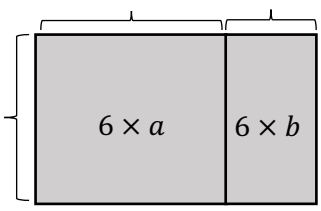
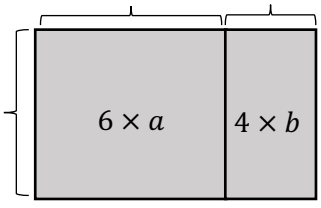


d)



Week 2 Lesson 4: Number talks

Concept Corner

associativity	20	10	40
2	8	property	100

Copy and complete:

To calculate 18×5, I could use _____ to rewrite the calculation as:

$$(9 \times \_\_) \times 5 = 9 \times (2 \times 5) = 9 \times \_\_ = 90$$

I could also use the distributive and commutative _____ to rewrite the calculation as:

$$5 \times (10 + \_\_) = 5 \times 10 + 5 \times 8 = 50 + \_\_ = 90$$

Or as:

$$5 \times (\_\_ - 2) = 5 \times 20 - 5 \times 2 = \_\_ + 10 = 90$$

1. **24×9** has been calculated in three different ways. Copy and complete the calculations. For each, draw a diagram to show the calculation and state which axiom has been used.

a) $24 \times 9 = 9 \times (20 + \_\_) = \_\_ \times 20 + 9 \times 4 = \_\_ + 36 = 216$

b) $24 \times 9 = (2 \times \_\_) \times 9 = \_\_ \times (12 \times 9) = 2 \times \_\_ = 216$

c) $24 \times 9 = 24 \times (\_\_ - 1) = 24 \times 10 - 24 \times \_\_ = 240 - \_\_ = 216$

d) $24 \times 9 = (4 \times 6) \times 9 = \_\_ \times (6 \times 9) = 4 \times \_\_ = 216$

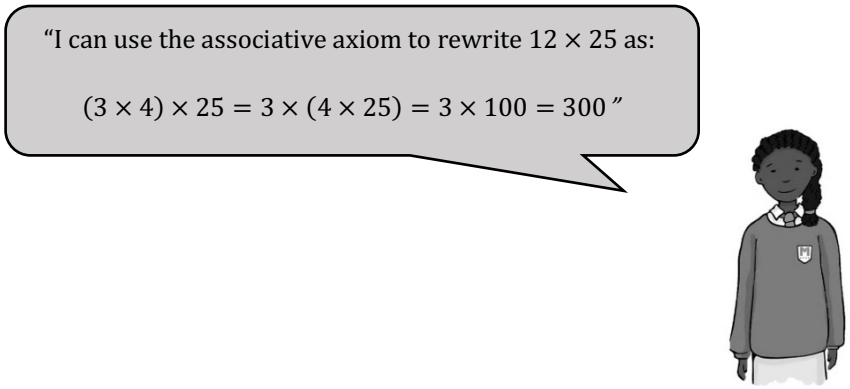
e) $24 \times 9 = 9 \times (10 + 10 + \_\_) = \_\_ \times 10 + \_\_ \times 10 + 9 \times \_\_ = 90 + \_\_ + \_\_ = 216$

2. **Without calculating the answer**, write out the calculation with either <, > or = between them to make statements correct:

14×5		14×6
24×9		23×8
18×5		9×10
17×7		16×8

3. Evaluate each of the following by first applying either the distributive or associative axioms. The first example has been done for you:

   a) 12×25
   b) 9×25
   c) 37×11
   d) 99×37
   e) 128×13
   f) $25 \times 16 \times 8$



4. Decide on an efficient calculation method and complete, showing the steps by writing calculations and/or drawing a model.

   a) 13×25
   b) 25×19
   c) 24×11
   d) 6×24
   e) 144×15
   f) $20 \times 12 \times 15$

Questions for depth

1. Rewrite each calculation below by applying the distributive axiom. Use this to evaluate each one.

   a) $8 \times 2.5 + 2 \times 2.5$ b) $93 \times 6.5 + 7 \times 6.5$
   c) $21 \times 0.3 + 79 \times 0.3$ d) $113 \times 1.87 - 1.87 \times 13$

2. For each of the following statements, decide if they are true or false. Justify your answer and give an example.

   a) Two numbers can be multiplied in any order
   b) All operations are commutative
   c) $\blacksquare \times (\blacktriangle + *) = (\blacktriangle + *) \times \blacksquare$
   d) $\blacksquare - (\blacktriangle - *) = (\blacksquare - \blacktriangle) - *$
   e) $\blacksquare \div (\blacktriangle + *) = \blacksquare \div \blacktriangle + \blacksquare \div *$
   f) Multiplication is distributive over addition and subtraction